

Name: Testing Testing  
 Spring 2026  
 Dr. Goadrich

## Rules

You may use your notes and the textbook, and the Typst documentation, but may not use the general internet or Large Language Models. Write up your answers in Typst and submit a PDF. Please draw any NFAs or DFAs using JFLAP, but do not use the extra conversion tools, I am expecting you to do the work yourself. Include any NFAs or DFAs drawn as images in your Typst document.

To COMPLETE this exam, you need to correctly answer **four** questions from Level 1, **one** question from Level 2, and an interpretation from Level 3. **Show all your work for full credit**

## Level 1

### 1. Regular Languages

Let  $\Sigma = \{0, 1\}$ .

$L$  = the number of 1s is odd and the number of 0s is a multiple of 4 and the word does not contain 10.

Show that  $L$  is regular by describing both a regular expression and a DFA.

### 2. Regular Closure

We define the middle( $L$ ) operation as follows

$$\text{middle}(L) = \{w : w, z, x \in \Sigma^*, |w| = |z| = |x|, zwx \in L\}$$

Determine if middle( $L$ ) is closed for regular languages.

### 3. Context-Free Language?

Is the following language context-free? Provide either a grammar, NPDA, or pumping lemma proof for your answer.

$$L = \{b^i a^j c^k a^j b^i c^m : i, j, k, m \geq 1\}$$

### 4. Another Context-Free Language?

Is the following language context-free? Provide either a grammar, NPDA, or pumping lemma proof for your answer.

$$L = \{a^i b^j c^i b^k a^i : i, j, k \geq 1\}$$

### 5. Turing Machine Creation

Design a Turing Machine that accepts the following language

$$L = \{1^i : i \text{ is a triangular number}\}$$

(see [https://en.wikipedia.org/wiki/Triangular\\_number](https://en.wikipedia.org/wiki/Triangular_number))

## Level 2

### 6. Turing Machine Processing

First, show the **instantaneous description** sequence of applying two strings to the Turing Machine described below, where the computation for one halts and the computation for the other does not.

$$Q = \{q_0, q_1, q_2, q_3, q_4\}$$

$$\Sigma = \{a, b\}$$

$$\Gamma = \{a, b, c\}$$

$$F = \{q_4\}$$

$$\delta(q_0, a) = (q_0, a, R)$$

$$\delta(q_0, c) = (q_4, b, L)$$

$$\delta(q_0, b) = (q_1, c, L)$$

$$\delta(q_1, a) = (q_1, a, R)$$

$$\delta(q_1, c) = (q_2, b, L)$$

$$\delta(q_2, a) = (q_3, a, R)$$

$$\delta(q_3, b) = (q_2, b, L)$$

Second, by definition, the language accepted by this Turing Machine is recursively enumerable. **Is it also recursive?** If so, provide a recursive Turing machine, otherwise, prove that no such recursive Turing Machine is possible.

### Turing Equivalence

Consider a model of a Turing machine in which each move permits the read-write head to travel two or three cells to the left or the right, the distance and direction of travel being one of the arguments to  $\delta$ . Give a **precise definition** of such an automaton and **show it is equivalent** to a standard Turing machine.

## Level 3

### Computation Interpretation

Use an alternative medium to interpret **an advanced concept** discussed in this course, such as P vs NP, NP-Completeness, Pumping Lemma, Universal Turing Machines, Halting Problem, or Nondeterministic Automata. Your medium could be a poem, piece of music, artwork, interpretive dance, epic rap battle, sculpture, existentialist film, Iron Chef-like meal, etc. (If you plan to use a medium besides something on this list, please check with me first.)

You are allowed to work in groups **ONLY ON THIS QUESTION** with the following stipulations:

1. The maximum group size is 4.
2. Everyone in the group must be an active participant in the creation and execution.
3. All group members must sign a document that details the contribution of each member.

You will have at most 15 minutes to present your creation during our final exam period, Monday May 11, at 9am.