

Name: Testing Testing

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Rules

You may use your notes and the textbook, and the Typst documentation, **but may not use the general internet or Large Language Models**. Write up your answers in Typst and submit a PDF. Please draw any NFAs or DFAs using JFLAP, but do not use the extra conversion tools, I am expecting you to do the work yourself. Include any NFAs or DFAs drawn as images in your Typst document.

To COMPLETE this exam, you need to correctly answer **three** questions from Level 1, and **one** question from Level 2. **Show all your work for full credit.**

Level 1

1. Context-Free Grammar

Write a context-free grammar that generates the language

$$L = \{a^n b^m c^q : n = m \text{ or } m = q \text{ where } n, m, q \geq 0\}$$

Is your grammar ambiguous? State your reasons behind your answer.

2. Grammar \rightarrow NPDA

Describe an NPDA which represents the same language as the grammar below:

$$\begin{aligned} S &\rightarrow Ax \\ A &\rightarrow aA \mid B \\ B &\rightarrow cc \end{aligned}$$

3. Improper Fractions

The set of all improper fractions can be written using strings n/d where n and d are positive integers, such that $n \geq d$. To represent this as a string, $\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, /\}$. For example 120/47, a string with 6 characters, is in this set. Determine whether or not this set is a context-free language.

4. DNF

Dhomsy Normal Form says all production rules must be one of two forms, $A \rightarrow x$ or $A \rightarrow BCD$, where x is a terminal symbol and A, B, C, D are variables. Can any context-free grammar be transformed into DNF? Either prove this statement true by providing an algorithm, or show a counter-example grammar that cannot be placed in DNF.

Level 2

5. CYK Parsing

Prove if the word $aaccc$ can be parsed or not by the following grammar using the CYK algorithm. If so, what is the parse tree for this word in the transformed CNF grammar? Show all of your steps.

$$\begin{aligned}S &\rightarrow AB \\A &\rightarrow aA \mid B \mid C \mid cc \\B &\rightarrow AcB \mid \lambda \\C &\rightarrow BC\end{aligned}$$

6. Pumping Lemmas

Determine if the following language is a Regular Language, a Context-Free Language, or neither.

$$L = \{0^i 1^j 2^k : k > j > i, i \geq 2\}$$