Name: _____

Rules

You may use your notes and the textbook, and the Typst documentation, but may not use the general internet or Large Language Models. Write up your answers in Typst and submit a PDF. Please draw any NFAs or DFAs using JFLAP, but do not use the extra conversion tools, I am expecting you to do the work yourself. Include any NFAs or DFAs drawn as images in your Typst document.

To COMPLETE this exam, you need to correctly answer **three** questions from Level 1, and **one** question from Level 2. **Show all your work for full credit**

Level 1

Comments

Many programming languages use special symbols to encapsulate comments. For example, Java uses /* and */ to open and close a single comment. There is a regular language that represents all properly formatted comments. For example, this is a single comment

- /*abaababababaab*/

But these are not

- /*bbbabbaba*/bbab*/

- /bab****//

- /*bbbabbaba*//*bbab*/

These examples are not comprehensive, think of other ways that comments can be incorrectly formatted.

Find a DFA that accepts this single comment language, using $\Sigma = \{a, b, /, *\}$.

Lemurs and Fossa

In Madagascar, two lemurs and two fossa must cross a river using a grass raft which can carry at most two animals. However, fossa are predators of lemurs. For both banks, if there are lemurs present on the bank, they cannot be outnumbered by fossa (if they were, the fossa would eat the lemurs). The raft cannot cross the river by itself with no animals on board. Using the alphabet $\Sigma = \{L, F, C, P, A\}$ to denote a Lemur traveling alone, a Fossa traveling alone, a Conspiracy of two lemurs traveling together, a Pack of fossa traveling together, and an Alliance of a lemur and a fossa traveling together, write an NFA to describe the language of valid solutions to this puzzle.
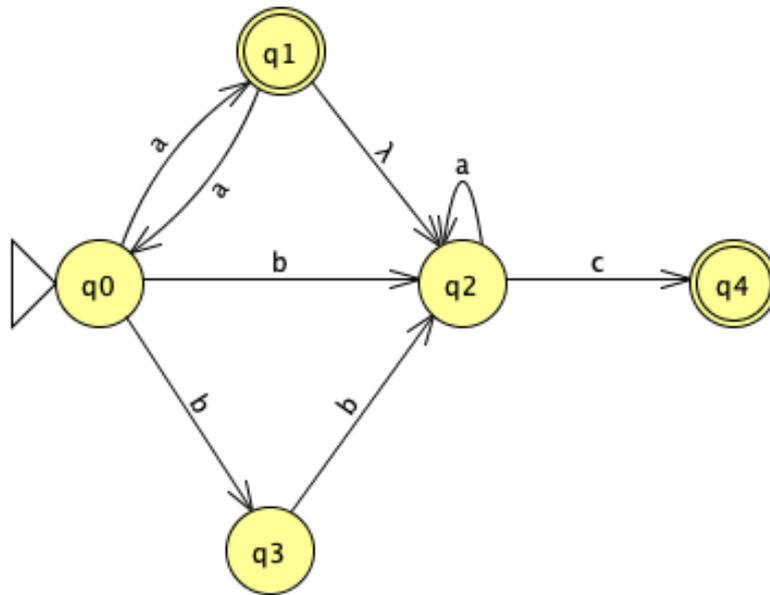
NFA → DFA

Figure 1: A happy little NFA

Convert the NFA in Figure 1 into a DFA following the algorithm discussed in class.

RE → NFA

Write an equivalent NFA for the language represented by the regular expression $0^*1(0 + 11)^*$.

Level 2

Order

A regular language L has an *order*, defined as the smallest integer n for which $L^n = L^{n+1}$. When this does not exists, the *order* is ∞.

1. Show that the order of any finite language is ∞.

2. What is the order of the language $(\lambda + (aaa)(aa)^*)$?

3. What is the order of the language $(\lambda + b^*a)(b + ab^*ab^*a)^*$?

Insert

Consider all words that can be formed from a given regular language L by inserting a new symbol anywhere into strings from L. For example, if $\Sigma = \{e, m, u\}$ and $L = \{emu\}$, then

$insert(L) = \{eemu, emeu, emue, memu, emmu, emum, uemu, eumu, emuu\}$

1. Formally define this operation *insert* for **all** languages.

2. Describe how to construct an NFA for $insert(L)$, given an NFA for L.