Name: _____

CSCI 380: Activity 3

Languages[1]

Team Roles	Team Member
Facilitator: reads the questions aloud, keeps track of time and makes sure everyone contributes appropriately.	
Spokesperson: talks to the instructor and other teams.	
Quality Control: records all answers & questions, and provides team reflection to team & instructor.	
Process Analyst: Considers how the team could work and learn more effectively.	

> **Note**
>
> If you have 3 people, combine Facilitator & Process Analyst.

Learning Objectives

1. Develop a formal definition for **strings** and their **operators**.
2. Understand how sets of strings can form **languages**.
3. Reason about **closure properties** of operators on languages.

In this Activity, you will find two Modules and one Reflection section.

[1] content derived from Introduction to Formal Languages and Automata by Peter Linz. https://www.jblearning.com/catalog/productdetails/9781284231601

start
time:

Model I. Strings (20 min)

An **alphabet** is finite, nonempty set of symbols, denoted with Σ. For example, $\Sigma = \{a, b\}$ is an alphabet with two symbols, a and b.

A **string** is finite sequence of symbols from a given alphabet. w, u, and v are common names for strings. For the above alphabet Σ, an example of a string is $w = abaaa$.

Explore the Model Questions

1. Write three other example strings that can be made from symbols in $\Sigma = \{a, b\}$.

2. Write three example strings from the alphabet $\Sigma = \{0, 1, 2\}$.

A few more pieces of notation will help expand our ability to discuss strings and alphabets.

- $w \cdot v$, or more simply wv, is the **concatenation** of two strings. Formally, if $w = a_1a_2a_3...a_n$, $v = b_1b_2b_3...b_m$, then $wv = a_1a_2a_3...a_nb_1b_2b_3...b_m$.
- w^R is the **reverse** of a string. If $w = a_1a_2a_3...a_n$, then $w^R = a_na_{n-1}...a_2a_1$
- $|w|$ is the **length** of the string. This is a count of the number of symbols in the string.

3. Let $w = ab, u = abb, v = aaa, x = aa$. How can you concatenate any combination of w, u, and v to create $t = abaaabbabaa$?

4. Let $w = aaabab$. Is $w^R = babbaa$? Why or why not? Be formal in your answer.

5. What are the lengths of w, u, v, x, and t in question 3 above?

Just some last bits of notation before we have enough for strings:

- We denote the **empty string** with λ. The empty string has no length, so $|\lambda| = 0$. Also, with respect to concatenation, $\lambda w = w\lambda = w$.
- A **substrings** of w is any sequential subsequence of symbols from w.
- For all decompositions of a string w where $w = uv$, u is a **prefix** of w and v is a **suffix** of w.
- w^n is the **exponent** operator, using repeated concatenation, meaning the string obtained by concatenating w with itself n times.

♣

6. How many substrings abb are in ww^Rw, where $w = aabbab$?

7. Let the string $w = aab$.

   a. What is the value of w^3?

   b. What is the value of w^0?

Critical Thinking Questions

8. For addition in $\mathbb{Z}$, we call 0 the identity element, because $\forall x \in \mathbb{Z}, 0 + x = x + 0 = x$. What is the identity element for concatenation in strings?

9. If $w = abba$ then $\{\lambda, a, ab, abb, abba\}$ is set of all prefixes of w. Why is λ in this set?

10. Prove the following statement using *induction*.

Proposition 1

$|w^n| = n\ |w|$

♠

STOP

Wait for all teams to complete this model. The spokesperson will be reporting out to the rest of the class.

start
time:

Model II. Languages (20 min)

Recall that Σ is an alphabet of symbols.

- Σ^* is the set of strings obtained by concatenating zero or more symbols from Σ.
- Sometimes we don't want the empty string, so we define $\Sigma^+ = \Sigma^* - \{\lambda\}$.
- A **language** is a subset of Σ^*. We commonly denote languages with L.

Explore the Model Questions

1. Let $\Sigma = \{a, b\}$. Decide whether the following strings are in Σ^* or not. Explain your answers.

   a. ab

   b. $aabbaa$

   c. $aaabbbccc$

   d. bob

   e. λ

2. What is $|\Sigma^+|$?

3. Restate the following languages by listing their contents.

   a. $\{a^n : n \text{ is less than } 40 \text{ and divisible by } 7\}$

   b. $\{a^n b^m : n < 3 \text{ and } m < 2\}$

4. Let $\Sigma = \{a, b\}$ and $L = \{a^n b^n : n \geq 0\}$. Decide whether the following strings are in the language or not. Explain your answers.

   a. ab

   b. $aabbaa$

   c. $aaabb$

   d. $aaabbb$

   e. λ

   f. $bbaa$

5. For the language defined in question 4, What is $|L|$?

Most interesting languages are infinite. Since languages are sets, the normal set operations of **union**, **intersection**, and **difference** will apply. The **complement** of a language is with respect to the domain of Σ^*.

We also extend some of the operations we learned on strings to apply to languages:

- The **reverse** of a language $L^R = \{w^R : w \in L\}$.
- The **concatenation** of two languages is the set of all strings obtained by concatenating any element of each. $L_1L_2 = \{xy : x \in L_1, y \in L_2\}$
- L^n is L concatenated with itself n times.

♣

6. Let $\Sigma = \{a, b\}$ and $L = \{a^n b^m : n, m > 0\}$. Decide whether the following strings are in L^R or not. Explain your answers.

   a. ab

   b. $aabbaa$

   c. $bbbaa$

   d. $aaabbb$

   e. λ

   f. $bbaa$

7. Let $\Sigma = \{a, b\}$ and $L_1 = \{a^n : n \geq 0\}, L_2 = \{b^m : m \geq 0\}$. Decide whether the following strings are in L_1L_2, L_2L_1, both, or neither. Explain your answers.

   a. abb

   b. $aabbaa$

   c. $bbbaa$

   d. $aaabbb$

   e. λ

   f. $bbaa$

8. Given $L = \{a, aa, ab\}$, write four elements in L^4 that have different lengths.

9. For all languages, what is L^0?

The **closure** of a set is a set of all possible elements that can be reached with a given operator. Two closures of languages are given here:

- $L^* = L^0 \cup L^1 \cup L^2 \ldots$ is the **star-closure** of a language L.
- As above with Σ^+, $L^+ = L^1 \cup L^2 \ldots$ is the **positive-closure** of a language L.

Critical Thinking Questions

10. Prove or disprove the following claim:

Proposition 1

$(L^R)^* = (L^*)^R$

STOP

Wait for all teams to complete this model. The spokesperson will be reporting out to the rest of the class.

Looking Back - Group Reflection

Review the job descriptions on your role card. Evaluate privately on how well you performed in your role. Provide the following using the **SII framework**:

S – *Strength* (Also what specifically did you do that would indicate that it was a strength)

I – *Improvement* Area

I – *Insight* concerning either the process or the content of the activity

Facilitator

List below how long each Model took your group:

- Model I:
- Model II:

Process Analyst

Give feedback to each of your other team members with a **strength** that you observed, either in their role or their process skills.

- Facilitator
- Quality Control
- Spokesperson

Quality Control

Summarize the activity for your group members, by answering the following questions.

1. What is one similarity between the strings you have used in programming languages and the strings as described here?

2. What is one difference between the strings you have used in programming languages and the strings as described here?

Spokesperson

Gather up the name cards and any other materials and give them to Dr. Goadrich.