

Substitution

$$[x \mapsto e_1] e_2$$

"Substitute e_1 for x in e_2 "
 e_2 , but with every occurrence of x replaced by e_1

Ex.

$$[y \mapsto S(SZ)] (\underline{y}) + (S(S\underline{y})) \cdot \underline{y}$$

$$\rightarrow ((S(SZ)) + (S(S(S(SZ)))) \cdot (S(SZ)))$$

Ex. Recall our syntax + rewrite system for arithmetic,
+ "let-expressions"

$$e ::= n \mid e + e \mid e - e \mid e \times e \mid \text{let } \overset{\text{any variable}}{x} = e_1 \text{ in } e_2$$

eg. $\text{let } x = 7 + 3 \text{ in } x + x$ ←

$3 + (\text{let } y = 8 \text{ in } (\text{let } z = y + y \text{ in } z - 2))$ ←

Q: how to evaluate? Std rewrite rules for +, -, ×, and:

$$(\text{let } x = \underline{n} \text{ in } \underline{e}) \longrightarrow [x \mapsto n]e$$

must be an integer

eg.

$$\text{let } x = \underline{7 + 3} \text{ in } x + x$$

$$\rightarrow \underline{\text{let } x = 10 \text{ in } x + x}$$

$$\rightarrow \underline{[x \mapsto 10](x + x)}$$

$$\rightarrow 10 + 10$$

$$\rightarrow 20$$

$$3 + \underline{(\text{let } y = 8 \text{ in } (\text{let } z = y + y \text{ in } z \cdot 2))}$$

$$\rightarrow 3 + [y \mapsto 8](\text{let } z = y + y \text{ in } z \cdot 2)$$

$$\rightarrow 3 + (\text{let } z = \underline{8 + 8} \text{ in } z \cdot 2)$$

$$\rightarrow 3 + \underline{(\text{let } z = 16 \text{ in } z \cdot 2)}$$

$$\rightarrow 3 + (16 \cdot 2)$$

$$\rightarrow 3 + 32$$

$$\rightarrow 35.$$