

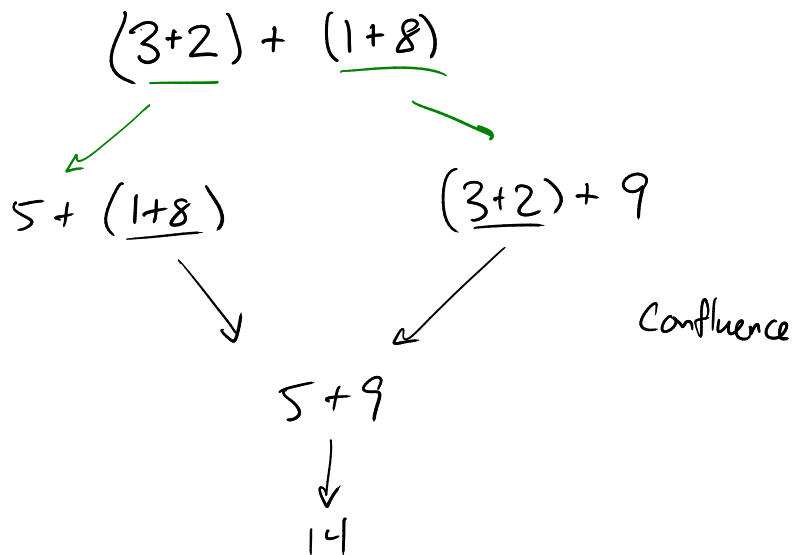
## Rewriting.

eg  $3 + ((2 - 5) \times 7)$

$\rightarrow 3 + ((-3) \times 7)$

$\rightarrow 3 + (-21)$

$\rightarrow -18.$



More generally: Syntax + Rewrite rules.

$$e ::= n \mid e_1 + e_2 \mid e_1 - e_2 \mid e_1 \times e_2$$

↑  
integer

Defining what possible expressions ("e") look like. We can use parentheses to disambiguate.

(BNF -  
Backus-Naur  
Form  
/"grammar")

eg.  $1 + (2 - 4)$  vs.  $(1 + 2) - 4$

Rewrite rules tell us when we can replace one expression by another.

eg.  $n_1 + n_2 \rightarrow n_3$  where  $n_3$  is the sum of  $n_1$  and  $n_2$

eg.  $3 + 2 \rightarrow 5.$

+ similar rules for  $-$ ,  $\times$ .

Typically, we can apply a rule anywhere inside an expression

eg.  $1 + (2 + 3) \rightarrow 1 + 5.$

g.

$$P ::= \textcircled{x} \mid \neg P \mid P_1 \vee P_2 \mid P_1 \wedge P_2$$

any variable

$$\text{g. } (x \wedge \neg y) \vee (\neg(y \wedge z))$$

Rewrite rules: (goal: rewrite into Conjunctive Normal Form).

$$\neg\neg P \rightarrow P$$

$$\neg(P \vee Q) \rightarrow \neg P \wedge \neg Q$$

$$\neg(P \wedge Q) \rightarrow \neg P \vee \neg Q$$

$$P \vee (Q \wedge R) \rightarrow (P \vee Q) \wedge (P \vee R)$$

$$\underline{(Q \wedge R) \vee P} \rightarrow (Q \vee P) \wedge (R \vee P)$$

$$(x \wedge \neg y) \vee (\neg(y \wedge z))$$

$$\rightarrow \underline{(x \wedge \neg y) \vee (\neg y \vee \neg z)}$$

$$\rightarrow \underline{(x \vee (\neg y \vee \neg z)) \wedge (\neg y \vee (\neg y \vee \neg z))}$$


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$$e ::= Z \mid Se \mid e_1 + e_2 \mid e_1 \cdot e_2$$

$$\text{g. } S(SZ)$$

$$(S(S(SZ)) \cdot S(SZ)) + (SZ)$$

Rewrite rules:

$$Z + e \rightarrow e$$

$$(Se_1) + e_2 \rightarrow S(e_1 + e_2)$$

$$e \cdot Z \rightarrow Z$$

$$e_1 \cdot (Se_2) \rightarrow e_1 + (e_1 \cdot e_2)$$