Lambda Calculus

Types for λ-calculus? → Simply typed λ-calc (STLC)

terms $t ::= x \mid \lambda x.\, t \mid t_1\ t_2 \mid n \mid +$

types $\tau ::= \mathbb{N} \mid \tau_1 \to \tau_2$

contexts (Gamma) $\Gamma ::= \cdot \mid \Gamma, x:\tau$ ← aka Map Var Type.

Rules for giving types to terms

"turnstile" \vdash

$$\frac{}{\Gamma \vdash + : \mathbb{N} \to \mathbb{N} \to \mathbb{N}}\ \text{PLUS}$$

$$\frac{}{\Gamma \vdash n : \mathbb{N}}\ \text{NAT}$$

$\Gamma \vdash t : \tau$

"In context Γ / given the assumptions in Γ, t has type τ".

$$\frac{P_1 \quad P_2 \quad P_3}{Q}$$

means $(P_1 \wedge P_2 \wedge P_3) \Rightarrow Q$.

$$\frac{\Gamma \vdash t_1 : \sigma \to \tau \qquad \Gamma \vdash t_2 : \sigma}{\Gamma \vdash t_1\ t_2 : \tau}\ \text{APP}$$

$$\frac{x:\tau \in \Gamma}{\Gamma \vdash x : \tau}\ \text{VAR} \quad \text{(aka lookup)}.$$

$$\frac{\Gamma, x:\sigma \vdash t : \tau}{\Gamma \vdash \lambda x.t : \sigma \to \tau}\ \text{LAM}$$